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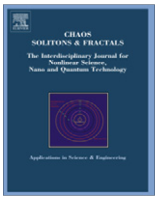

Exact thermodynamic principles for dynamic order existence and evolution in chaos

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ABSTRACT

The *negentropy* proposed first by Schroedinger is re-examined, and its conceptual and mathematical definitions are proposed. This re-definition of negentropy integrates Schroedinger's intention of its introduction, and the subsequent diverse notions in literature. This negentropy is further corroborated by its ability to state the two exact thermodynamic principles: *negentropy principle* for dynamic order existence and *principle of maximum negentropy production* (PMNEP) for dynamic order evolution. These principles are the respective counterparts of the existing *entropy principle*, and the *law of maximum entropy production*, respectively. The PMNEP encompasses the basic concepts in the evolution postulates by *Darwin* and *de Vries*. Perspectives of dynamic order evolution in literature point to the validity of PMNEP as the *law of evolution*. These two additional principles now enable unified explanation of order creation, existence, *evolution*, and destruction; using thermodynamics.

1. Introduction and review

The laws that govern an organization are derived from the two laws of thermodynamics. The first law states that the total *quantity* of energy in an isolated system remains constant. The 2nd *law of thermodynamics* (also called the *entropy principle* (EP)) came in to existence after the 1st *law* was established. The 'entropy, S' refers to the diminished magnitude of finite gradients of field variables, $|\mathrm{Grad}(F)|_g$, in an isolated system; and is the classical signpost of natural change [1]. The EP is a restatement of a unifying law of nature, which implies that on a global basis (i.e. in the net), magnitudes of gradients of field variables are always diminished. Diminishing $|\mathrm{Grad}(F)|_g$ leads to dispersion of mass and/or energy leading to reduction in predictability, i.e. disorder (entropy) increase; thus, $S\uparrow \Leftrightarrow |\mathrm{Grad}(F)|_g\downarrow$. The definitions of other specific thermodynamic terms used in this investigation are listed in Table 1. An issue that has mystified even pioneering researchers [2,3] is the creation, existence, and evolution of dynamically ordered structures in increasing disorder in chaotic surroundings. The prime reason for this incomplete understanding is because exact physical laws have not been identified for ordering, similar to exact laws that govern disorder.

1.1. Review on definitions and interpretations of negentropy

The '*negentropy*' first came from Schroedinger [4], who stated: 'what an order feeds upon is negentropy; thereby succeeding in freezing itself from the entropy it cannot help producing'. This statement suggests that the process of negentropy generation for order existence is responsible for localised control of entropy increase. However, in a footnote later, Schroedinger

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Nomenclature

E	energy content in dynamic order (J)
$\dot{E}$	energy flow rate into and out of dynamic order (W)
F	symbol for field variable/s
J	symbol for rate/s of irreversible process/es
m	mass (kg)
$\dot{m}$	mass flow rate into and out of dynamic order (kg/s)
N	number of ordered sub-systems at time instant t (–)
S	entropy (J/K)
$\dot{S}$	rate of entropy flow into/out of dynamic order (W/K)
$\ddot{S}$	2nd derivative of S w.r.t. time ($= \mathrm{d}\dot{S}/\mathrm{d}t = \mathrm{d}^2S/\mathrm{d}t^2$) (W/s K)
s	specific entropy per unit mass (J/kg K)
$s^{\#}$	specific entropy per unit energy (K^{-1})
$\dot{s}$	rate of specific entropy change (W/kg K)
$\dot{S}_{\mathrm{gen}}$	rate of entropy generation (W/K)
$s_{\mathrm{n}i}$	negentropy of ith ordered sub-system (J/kg K)
$\dot{s}_{\mathrm{n}i}$	rate of negentropy change of ith dynamic order (W/kg K)
t	time instant (s)

Subscripts

c	creation of dynamic order
D	destruction of dynamic order
d	surroundings (disorder relative to ordered sub-systems)
E	energy exchange of dynamic order with surroundings
e	existing ordered sub-system
ex	leaving ordered sub-system
g	global i.e. for complete isolated system
i, j, k	ith, jth, kth ordered sub-system, respectively
in	entering ordered sub-system
irr	irreversibilities not related to dynamic order
IS	isolated system
m	mass exchange of dynamic order with chaos
max	maximum value
o	dynamic order
T	total value for isolated system
thr	threshold value above which dynamic order begins to merge with chaos

Superscripts

*	state in which all dynamic order is replaced by chaos of same mass
(i), (ii)	Cases (i), (ii)

Special Abbreviations

EP	entropy principle
IP	inflection point (Ref. Fig. 1)
LMEP	law of maximum entropy production
NEP	negentropy principle
PMNEP	principle of maximum negentropy production
TMEP	theorem of minimum entropy production

explains that *negentropy* is free *energy*, which can be harnessed for ordering. Later, Ho [5] explained that *negentropy* is mobilisable stored *energy* in a self-organized system. Due to the lack of a universal definition of negentropy, directives for dynamic order creation and existence were stated by Mahulikar and Herwig [6] using '*entropy*'.

1.2. Review on fractal-based scaling and universality of order

Spencer [7] stated that biological evolution is part of universal process of evolution, because evolution pervades the inorganic as well as the organic realm. Spencer's work also treated 'super-organic evolution' (social evolution), and evolution of 'super-organic products' (cultural evolution) [8]. Damiani and Franca [9] discussed that ordered patterns are fractal-based; and exist at the boundary between order and chaos, and evolve by increasing their complexity. Wu et al. [10] stated that the

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Table 1
Definitions of specific thermodynamic terms used in this investigation

Terms	Definitions
Global equilibrium	State of the isolated system when the magnitudes of gradients of all field variables in the isolated system are zero
Static order	Ordered sub-systems that can be created in the vicinity of global equilibrium based on minimisation of free energy, e.g. crystals [25]
Dynamic order	Localised self-organisation in an unstable isolated system, which can exist far from global equilibrium; by default, *order* refers to *dynamic order*. It acquires a spatial, temporal, or functional structure without specific interference from its surroundings in a non-specific manner [24]
Local gradient of field variable	$\lvert\mathrm{Grad}(F)\rvert_{oi}$: Magnitude of gradients of field variable across the boundary between dynamic order *oi* and its surrounding chaos
Local equilibrium	Equilibrium between *ordered sub-systems* and chaos, which is the condition when *dynamic order* is completely converted into disorder; and $\lvert\mathrm{Grad}(F)\rvert_{oi} = 0$
Total disorder	State of the isolated system that has no dynamic order, i.e. only disorder; this state can also exist far from global equilibrium
Global gradients of field variables	$\lvert\mathrm{Grad}(F)\rvert_{g} = 0$: Representative of magnitude of net gradients of field variables in the entire isolated system
Maximum disorder	The state of global equilibrium in the isolated system is a state of *maximum disorder*, i.e. maximum entropy of the isolated system ($S_{IS} = S_{IS,max}$). In this state, $\lvert\mathrm{Grad}(F)\rvert_{g} = 0$ and $\dot{S}_{IS} = 0$; it is one of the states of *total disorder* because dynamic order cannot exist (only static order can exist)
Evolutionary entropy [19]	It is a statistical measure of the variability in the age of reproducing individuals in a population, and characterises the complexity of the life cycle and population stability
Malthusian parameter	Constant in the exponent of exponential equation of population growth

universe is essentially fractal-based on the scale of galaxies and their clusters. Azbel' [11] also discussed universality in evolutionary origin and the need for a multi-disciplinary study. Grigolini et al. [12] discussed classical and quantum complexity in light of non-extensive thermodynamics, which provides the desired power laws. This approach was suggested due to an overlap in the physical descriptions of complex and non-extensive systems.

1.3. Review on order evolution

Spencer [7] defined *evolution* as a process of the transformation of less ordered to more ordered states, following from an unidentified exact natural law. Lately, Salthe [13] defined evolution as the irreversible accumulation of the effects of historical contingency. A popular evolution postulate is by Darwin [14], which begins with a hypothesis that life on Earth arose from non-living matter. Thereafter, life proceeded to evolve into more complex forms, by random mutation and *natural selection* process (popularly known as *survival of fittest*). The other well-known evolution postulate is the *mutation theory* [15], which states that mutations alone can bring about the abrupt and noticeable evolutionary changes in dynamic order. The evolution of ecological systems shows a tendency to maximize energy flows by configurations and processes at several levels [16]. Elitzur [17] stated that the issue of how life emerged from inanimate matter yields novel insights when discussed in the light of thermodynamics. A model was proposed based on the assumption that life began with the accidental assembly of a self-replicating molecule. The evolution of self-replicating systems was shown by Elitzur to be highly efficient in extracting, recording, and processing information about the environment.

An increase in the number of genetic microstates in a population increases the entropy of the population's information. The maximum evolutionary potential of a population is the maximum number of possible microstates [18]. The evolutionary entropy across species is a measure of the variability in the age of reproducing individuals. The evolutionary entropy [19] of a particular species constitutes the operationally valid measure of Darwinian fitness (relative probability of survival and reproduction for a genotype). Demetrius [19] showed that evolutionary entropy rather than the Malthusian parameter represents a non-equilibrium analogue of thermodynamic entropy in EP. Corbet [20] explored a model of pre-biotic system for a general understanding of evolution based on thermodynamics. The *evolutionary potential* [18] that incorporates entropy effects via EP, was shown as a necessary condition for the ordering of living systems. Evolution produces more varieties of ordered matter while also increasing complexity, which is measured based on Kolmogorov's algorithmic complexity or from information theory [21]. Michaelian [22] states that evolution through natural selection is a manifestation of non-equilibrium thermodynamic derivatives. Eigen and Schuster [23] proposed the model of hypercycles as a hypothetical stage of macromolecular evolution, which could follow quasi-species.

1.4. Inferences from review

Based on review on the role of thermodynamic principles in explaining dynamic order existence and evolution, the following important inferences are drawn:

i. There is no universally accepted definition of *negentropy* that enables explanation of order origin, existence, and evolution; by exact negentropy-based thermodynamic principles.

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ii. There is universality in evolutionary origin, selection, and mortality, which extends from biology to cosmology and sociology also. This inference follows especially based on the definitions of evolution proposed from 1st principles, e.g. by Salthe [13].
iii. Dynamics of order, which include creation, existence, evolution, and destruction at all scales share the common fractal, whose patterns are governed by thermodynamic principles.
iv. Thermodynamics enables dealing with arbitrary and complex ordered sub-systems from a universal point of view [24].

1.5. Objectives and scope

This investigation is based on the probabilistic aspects of non-linear dynamical systems that are the foundations of statistical mechanics and chaos. Thermodynamics is the macroscopic outcome of statistics of mechanics at the microscopic level. The quest for exact phenomenological non-equilibrium thermodynamic principles for dynamic ordering restricts the scope of this investigation to analysis of macroscopic phenomena.

The two assumptions made regarding the isolated system that has dynamically ordered sub-systems are elaborated below. These assumptions define the scope of this investigation, and are the conditions for dynamic order to be created and to co-exist with chaos [25].

Assumption (i): The isolated system is embedding; i.e. it can embed open systems, e.g. the dynamically ordered sub-systems. An example of an embedding isolated system to which the laws of thermodynamics can be applied, is the system comprising of the earth as an open system. The closed system comprising of earth, the sun that feeds it with energy and part of the universe that receives the energy emitted by earth at night, can be treated as an isolated system [24].

Assumption (ii): As illustrated in Fig. 1

$$\ddot{S}_{\mathrm{IS}} > 0, \tag{1}$$

i.e. the isolated system (IS) is sufficiently far from equilibrium for it to be dynamically unstable and chaotic. Dynamic order and its evolution in chaos is a manifestation of this thermodynamic instability (resulting from stabilisation of fluctuations by mass and energy exchange [25]). As the isolated system approaches equilibrium, the sign of $\ddot{S}_{\mathrm{IS}}$ changes at a particular inflection point (IP in Fig. 1); and thereafter

$$\ddot{S}_{\mathrm{IS}} < 0. \tag{2}$$

The entropy–time diagram of an isolated system in Fig. 1 is constructed based on the satisfaction of inequality (1) before (2), in time. To the left and bottom of IP (III-quadrant), i.e. earlier in time, the isolated system follows inequality (1). The path taken by an open system (e.g. dynamic order) within the isolated system is governed by the law of maximum entropy production [26] (LMEP, refer Table 2). As per LMEP, stable existence of localised dynamic ordering is justified by its role in increasing global entropy production (termed as 'dissipative structures' [25]). The LMEP is based on the preference for path of least resistance, for minimizing the net magnitude of global gradients of field variables, $|\mathrm{Grad}(F)|_{\mathrm{g}}$.

The EP [1] and LMEP [26] are ubiquitous and exact physical laws that determine spontaneity and its rate, i.e. disorder increase and its rate, respectively. Dewar [27] applied Jayne's' information theory formalism of statistical mechanics to the stationary states of open, non-equilibrium systems. The LMEP was derived as the selection principle for stationary non-equilibrium states, and the emergence of self-organised criticality was derived for flux-driven systems. Later, Dewar [28] presented a mathematical derivation of LMEP by showing that the fluctuation theorem [29] allows a general orthogonality property from which, LMEP follows. The rate at which $|\mathrm{Grad}(F)|_{\mathrm{g}}$ is diminished is the *degree of spontaneity*, i.e. the equivalent of rate of entropy change, $\dot{S}_{\mathrm{IS}}$. Therefore, '$\dot{S}_{\mathrm{IS}} > 0$' is a spontaneous process; and '$\dot{S} < 0$', which is possible only locally as per the fluctuation theorem [29], is a local non-spontaneous process. Occurrence of localised non-spontaneous

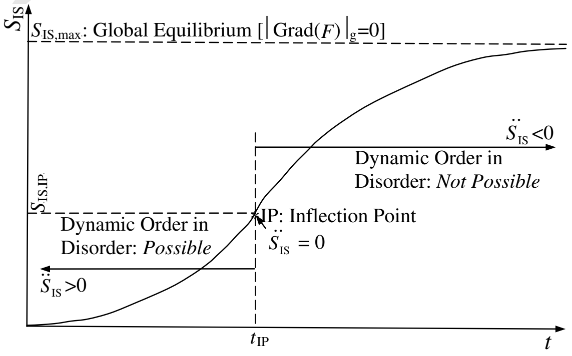

Fig. 1. Illustration of regimes on entropy–time diagram for isolated system (IS).

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Table 2
Thermodynamics directives cited for explaining operations of dynamic order

Directive	Statement
Law of maximum entropy production (LMEP) [26]	The isolated system will select the path or assemblage of paths out of available paths that minimises the potential or maximizes entropy of the isolated system (S_{IS}) at the fastest rate for given constraints
Theorem of minimum entropy production (TMEP) [30]	The variation of $\dot{S}_{\mathrm{IS}}$ is negative or zero in non-equilibrium stationary states in the vicinity of equilibrium
Slaving principle [24]	Superior ordered sub-systems have a tendency to govern the behaviour of inferior ones, during their co-existence

processes (e.g. order existence and evolution) satisfy EP, because they result in net positive *degree of spontaneity* ($\dot{S}_{\mathrm{IS}} > 0$). However, LMEP further implies that localised non-spontaneous processes are paths that result in net higher *degree of spontaneity* than exclusively by spontaneous processes. Therefore, existence and evolution of order are paths of lesser resistance relative to disorder only, for diminishing $|\mathrm{Grad}(F)|_{\mathrm{g}}$ faster. For given $|\mathrm{Grad}(F)|_{\mathrm{g}}$, minimising resistance leads to maximising the rate/s of irreversible process/es J; i.e. $\dot{S}_{\mathrm{IS}} = J\cdot \mid \mathrm{Grad}(F)|_{\mathrm{g}}$, is maximised.

To the right and top (I-quadrant) of IP, S_{IS} increase gradually approaches the asymptote; and the isolated system is dynamically stable (not chaotic) as per inequality 2. In this regime, the system characteristics are determined by its inertia; which prevents the system from reaching global equilibrium [$\dot{S}_{\mathrm{IS}} = 0$ and $|\mathrm{Grad}(F)|_{\mathrm{g}} = 0$]. Therefore, the system settles down to the tendency based on the *theorem of minimum entropy production* [30] (refer Table 2 for TMEP). In this regime, static order e.g. crystals can exist; whose objective is to freeze localised entropy production, thereby further reducing $\dot{S}_{\mathrm{IS}}$. The TMEP is valid in the framework of a strictly linear theory in which the deviations from equilibrium are so small that the Onsager reciprocity relations [31] are applicable. Localised dynamic ordering that further increase $\dot{S}_{\mathrm{IS}}$ is not supported by the dynamically stable surroundings (as fluctuations are dampened). These dissipative structures (that generate significant entropy in chaos) can exist and evolve far from global equilibrium, where the behaviour is the opposite of that indicated by TMEP [25]. Thus, existence and evolution of dynamic order in chaos satisfy inequality (1), and are feasible only to the left of IP in Fig. 1. This explains the existence and evolution of cosmological order only during the acceleration-phase of universe expansion [32].

2. 'Negentropy' re-defined

The re-definition of *negentropy* is proposed to enable direct explanation of order origin, existence, evolution, and destruction; by negentropy-based principles. The *negentropy* should encompass the following perspectives, with which it is equitably associated in literature:

(i) It should have *negative* sign, but its implication/s should extend beyond entropy or specific entropy with a negative sign;
(ii) it should have units either of *entropy* or specific *entropy*, but not of energy so that its nomenclature is valid;
(iii) it should enable quantitative accounting for the existence [4] and evolution of dynamic order in chaos;
(iv) because $S \geqslant 0$, negentropy is necessarily a *relative* measure of deviation from equilibrium of ordered sub-system with respect to chaos.

2.1. Re-definition of negentropy ($s_{\mathrm{n}i}$) of dynamically ordered sub-system (oi)

The following mathematical definition of negentropy of *i*th ordered sub-system is proposed:

$$s_{\mathrm{n}i} = s_{\mathrm{o}i} - s_{\mathrm{d}}. \tag{3}$$

The $s_{\mathrm{o}i}$ is specific entropy of *i*th order, and s_{d} is specific entropy of chaos; hence, negentropy is '*the specific entropy deficit of ordered mass with respect to its surrounding chaos*' ($s_{\mathrm{o}i} \ll s_{\mathrm{d}} \Rightarrow s_{\mathrm{n}i} \ll 0$). In cosmological and nuclear reaction studies, inter-convertibility of mass and energy is an important consideration. Therefore, mass is to be considered as a highly ordered form of energy; and the applicable specific entropy, $s^{\#}$, is based on per unit energy, with units, $(s^{\#}) = K^{-1}$. The $s^{\#}$ is also applicable when dealing with entropy changes associated with energy transfer; for instance, the increase in radiation-entropy during photosynthesis reaction in plants [33]. The definition of negentropy that integrates mass with energy based on ordering, and also between different levels of disorder in energy transfer is given as, $s^{\#}_{\mathrm{n}i} = s^{\#}_{\mathrm{o}i} - s^{\#}_{\mathrm{d}}$. The $s^{\#}_{\mathrm{n}i}$ is '*the specific entropy deficit of ordered energy with respect to the surrounding chaotic energy*'. In this investigation, specific problems dealing with inter-convertibility of mass and energy will not be addressed; therefore, $s_{\mathrm{n}i}$ based on Eq. (3) is used henceforth.

The $s_{\mathrm{n}i}$ reduces to zero when order o*i* fully integrates with surrounding chaos; therefore, $s_{\mathrm{n}i}$ is a measure of contrast of ordered sub-system *relative* to chaos. Since

$$S_{\mathrm{IS}} = \sum_{i=1}^{N_{\mathrm{e}}} m_{\mathrm{o}i}\cdot{}_{\mathrm{o}i} + m_{\mathrm{d}}\cdot s_{\mathrm{d}}, \quad \text{and} \quad m_{\mathrm{IS}} = \sum_{i=1}^{N_{\mathrm{e}}} m_{\mathrm{o}i} + m_{\mathrm{d}}, \quad s_{\mathrm{IS}} = s_{\mathrm{o}i} - s_{\mathrm{n}i} + \left(\sum_{i=1}^{N_{\mathrm{e}}} m_{\mathrm{o}i}\cdot s_{\mathrm{n}i}\right)\Big/ m_{\mathrm{IS}}; \tag{4}$$

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where, N_e is the number of ordered sub-systems existing. The proposed s_{ni} is now shown to provide exact principles for order existence and evolution in chaos, and the increase/decrease of s_{ni} in these principles are its absolute value ($|S_{ni}|$).

3. The negentropy principle (NEP)

Statement of NEP: '*For dynamic order to exist in chaos* (*or when dynamic order exists*), *its negentropy must increase*' ($|\dot{s}_{ni}| > 0 \Leftarrow \dot{s}_{ni} < 0$).

Conversely, 'when order o*i* ceases to exist, its negentropy begins to decrease' i.e. $|\dot{s}_{ni}| < 0 (\Leftarrow \dot{s}_{ni} > 0)$ until $s_{ni} = 0$. The last term on the right hand side of Eq. (4) is negligible, because $(m_{oi}/m_{IS}) \to 0$ and $\left(\sum_{i=1}^{N_e} m_{oi}/m_{IS}\right) \to 0$; therefore, increasing s_{IS} implies the following two exclusive cases:

(a) The – s_{ni} must increase (i.e. $|\dot{s}_{ni}| > 0$) and NEP holds, i.e. dynamic order exists in chaos; e.g. – s_{ni} increases when s_{oi} is maintained because s_d increases.
(b) If – s_{ni} reduces or remains the same, converse of NEP holds, and order *oi* is converted in to disorder.

The NEP can also be proved directly by differentiating equation (3) w.r.t. time as, $\dot{s}_{ni} = \dot{s}_{oi} - \dot{s}_d$; if order *oi* exists, its specific entropy is maintained relative to chaos. Therefore, $\dot{s}_{oi} \ll \dot{s}_d$; i.e. practically, $\dot{s}_{oi} = 0$, and $\dot{s}_{ni} = -\dot{s}_d = -\dot{S}_d/m_d$, and $\dot{S}_d = \dot{S}_{IS} - \frac{d}{dt}\left(\sum_{i=1}^{N_e} m_{oi} \cdot s_{oi}\right) = \dot{S}_{IS}$.

Thus

$$\dot{s}_{ni} = -\dot{S}_{IS}/m_d; \tag{5}$$

because, $\dot{S}_{IS} > 0$, $|\dot{s}_{ni}| > 0$, i.e. EP ⇒ NEP, when order exists in chaos.

For dynamic order o*i* to co-exist with chaos, $s_{oi} \leqslant s_{oi,thr}$; i.e. the specific entropy of order o*i* must be lower than a threshold [6]. This inequality leads to the sufficiency condition in conjunction with the earlier necessary condition, *Assumption (ii)*, for order o*i* to co-exist with chaos: $|\text{Grad}(F)|_{oi} \leqslant |\text{Grad}(F)|_{oi,thr}$. The $|\text{Grad}(F)|_{oi,thr}$ is the threshold magnitude of gradient of field variable across order o*i*, which it can withstand. If $\text{Grad}(F)|_{oi} > \text{Grad}(F)|_{oi,thr}$, order o*i* is destroyed, because its specific entropy then begins to exceed $s_{oi,\,thr}$. Since, the global gradients of field variables are diminished to a lower value due to existence of dynamic order, to satisfy EP

$$|\text{Grad}(F)|_g < [|\text{Grad}(F)|_g]^*. \tag{6}$$

The superscript '*' in the above equation denotes the state of total disorder in which all dynamic order is replaced by chaos of the same mass. This inequality results from the need to compensate for gradients of field variables maintained across dynamic order, to satisfy EP. These maintained gradients reduce localised entropy production by reducing *J*; hence, inequality Eq. (6) is an outcome of the inequality: $S_{IS} > S^*_{IS}$, where, $S^*_{IS} = m_{IS} \cdot s^*_{IS}$.

Dynamic order is an unstable open system [24] that is maintained in its identifiable state by a stable influx of energy and/or matter. Fig. 2 is a construction of the thermodynamic operation of dynamic order of mass m_{oi}, energy content E_{oi}, and specific entropy s_{oi}, in chaos (s_d). The order o*i* restricts its specific entropy s_{oi} increase (i.e. $\dot{s}_{oi} \to 0$) by interaction with chaos, in the form of mass/ energy exchange. Mass enters the ordered sub-system at flow rate $\dot{m}$, and an associated inlet entropy rate $\dot{S}_{m,in} (= \dot{m} \cdot s_{m,in})$; similarly, energy ($\dot{E}$) can flow in with inlet entropy rate $\dot{S}_{E,in} (= \dot{E} \cdot s^{\#}_{E,in})$. The variations in s_{oi} are much smaller than the increase in s_d; and variations in m_{oi} and E_{oi} are much smaller relative to $\dot{m}$ and $\dot{E}$, respectively. Therefore, s_{oi}, m_{oi}, and E_{oi}, are assumed constant and invariant over small observation time intervals; and the same $\dot{m}$ and $\dot{E}$ leave the ordered sub-system. However, their respective exit specific entropy rates are now much higher, i.e. $S_{m,ex} \gg S_{m,in}$, and $s^{\#}_{E,ex} \gg S_{E,in}$. These inequalities satisfy EP by compensating for localised ordering, in addition to the entropy generation due to irreversibilities along the flow of mass and energy. Order o*i* maintains the gradients of field variables across its boundary, $|\text{Grad}(F)|_{oi}$ [$\leqslant |\text{Grad}(F)|_{oi,thr}$], by this exchange of mass/energy. More the entropy generation in chaos due to mass/energy exchange, higher the values of $[|(\text{Grad}F)|_{oi}$ that can be maintained, because $|\text{Grad}(F)|_{oi,thr}$ is augmented. Thus, high s_{ni} (increased ordering) also results in high $|\dot{s}_{ni}|$ (increased rate of ordering), and vice versa also holds, i.e. $s_{ni} \uparrow \iff |\dot{s}_{ni}| \uparrow$.

The total entropy generation rate at time instant *t* in the isolated system in which order exists is given as

$$\dot{S}_{gen,T}(t) = \dot{m} \cdot [S_{m,ex}(t) - S_{m,in}(t)] + \dot{E} \cdot [S^{\#}_{E,ex}(t) - S^{\#}_{E,in}(t)] + \dot{S}_{gen,irr}(t). \tag{7}$$

The last term, $\dot{S}_{gen,irr}(t)$, is entropy generation due to irreversibilities in the isolated system that are not related to dynamic ordering. For illustrating *negentropy debt*, two cases without dynamic order existence are considered:– Case (*i*) with same

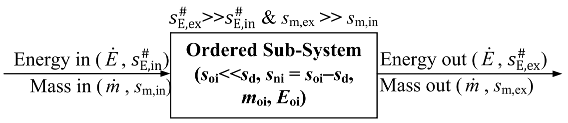

Fig. 2. Thermodynamic representation of dynamic order existence.

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mass/energy exchange as with order existing, and Case (*ii*) without mass/energy exchange. In both cases (*i*) and (*ii*), order *oi* is replaced by chaos of same mass (m_{oi}), but at much higher specific entropy $[S^*_{oi}(=s_d) \gg s_{oi}]$. In case (*i*), the inlet specific entropies associated with mass and energy interaction are considered the same as that with order existing, for reference. But now the exit specific entropies would be much lower than the case when the same energy and mass flow through dynamic order *oi*; i.e. $S^*_{m,ex} \ll S_{m,ex}$ and $S^{\#*}_{E,ex} \ll S^{\#}_{E,ex}$. The total entropy generation rate without dynamic order existing at same *t* for Case (*i*) is now given as

$$\dot{S}^{*(i)}_{gen,T}(t) = \dot{m}\cdot[S^*_{m,ex}(t) - S_{m,in}(t)] + \dot{E}\cdot[S^{\#*}_{E,ex}(t) - S^{\#}_{E,in}(t)] + \dot{S}_{gen,irr}(t) - m_{oi}\cdot\dot{S}_{ni}(t). \tag{7.1}$$

The total entropy generation rate without dynamic order existing at same *t* for Case (*ii*) is given as

$$\dot{S}^{*(ii)}_{gen,T}(t) = \dot{S}_{gen,irr}(t) - m_{oi}\cdot\dot{s}_{ni}(t); \tag{7.2}$$

where, $\dot{S}_{gen,T}(t) > \dot{S}^{*(i)}_{gen,T}(t) > \dot{S}^{*(ii)}_{gen,T}(t)$. The term present in Eqs. (7.1) and (7.2) (for state of total disorder) but absent in Eq. (7) (for state of order in chaos) is,

$$d/dt[m_{oi}\cdot s_d(t)] = -m_{oi}\cdot\dot{S}_{ni}(t) > 0. \tag{7.3}$$

Subtracting Eq. (7.1) from Eqs. (7) and (7.2) from Eqs. (7)–(9) are respectively obtained as follows:

$$\dot{S}_{gen,T}(t) - \dot{S}^{*(i)}_{gen,T}(t) = \dot{m}\cdot[s_{m,ex}(t) - s^*_{m,ex}(t)] + \dot{E}\cdot[s^{\#}_{E,ex}(t) - s^{\#*}_{E,ex}(t)] + [m_{oi}\cdot\dot{s}_{ni}(\boldsymbol{t})]; \tag{8}$$

and

$$\dot{S}_{gen,T}(t) - \dot{S}^{*(ii)}_{gen,T}(t) = \dot{m}\cdot[s_{m,ex}(t) - s_{m,in}(t)] + \dot{E}\cdot[s^{\#}_{E,ex}(t) - s^{\#}_{E,in}(t)] + [m_{oi}\cdot\dot{s}_{ni}(t)]; \tag{9}$$

The last term in square bracket in Eqs. (8) and (9), $[m_{oi}\cdot\dot{s}_{ni}(t)]$, is negative (refer Eq. (7.3)), and the left hand sides are positive. Thus, the first two positive terms on the right hand side more than compensate for the last negative term, due to entropy generation at a faster rate by order (as per LMEP). This compensation is especially due to exchange of mass/energy when order exists, i.e. $[\dot{S}_{gen,T}(t) - \dot{S}^{*(ii)}_{gen,T}(t)] > [\dot{S}_{gen,T}(t) - \dot{S}^{*(i)}_{gen,T}(t)]$. This compensation for the negentropy term in Eqs. (8) and (9) is the *negentropy debt*, which is continuously paid by dynamic order to chaos. The s_{oi} level is maintained locally by continuously draining out entropy to chaos, which is the sustenance of the fleeting disequilibrium [34]. Therefore, sustainable dynamic orders like flames are permitted to exist away from equilibrium with chaos, because they feed on negentropy to chaos. Because ordered sub-systems produce entropy at a rate sufficient to compensate for their internal ordering, the balance equation based on EP is not violated [4].

The total entropy generation rate in the isolated system at time *t* is given as

$$\dot{S}_{gen,T}(t) = -\sum_{i=1}^{N_e} m_{oi}(t)\cdot\dot{s}_{ni}(t) + \sum_{j=1}^{N_c}\dot{S}_{gen,c,oj}(t) + \sum_{k=1}^{N_D}\dot{S}_{gen,D,ok}(t) + \dot{S}_{gen,irr}(t). \tag{10}$$

The 1st term on the right hand side is entropy generation due to existence of N_e ordered sub-systems, 2nd is due to creation of N_c sub-systems, and 3rd due to destruction of N_D sub-systems. Since, $\dot{S}_{gen,T}(t)$ is maximised (by LMEP), creation, existence, and destruction of dynamic order, are the paths taken by the isolated system to maximise $\dot{S}_{gen,T}(t)$. Dynamic order evolution is included in the order existence term: $-\sum_{i=1}^{N_e} m_{oi}(t)\cdot\dot{s}_{ni}(t)$.

Over geological time intervals, the probability of a mismatch between evolution and gradients of field variables across dynamic order $|\text{Grad}(F)|_{oi}$ increases. As a result, the probabilistically created high $|\text{Grad}(F)|_{oi}$ $[\gg|\text{Grad}(F)|_{oi,thr}]$, destroy dynamic order. Extinction of a particular species [characterised by a band of $|\text{Grad}(F)|_{oi,thr}$] is due to their inability to avoid high $|\text{Grad}(F)|_{oi}$. Mortality of order is an instrument of natural selection and biological diversity [11], which also maximises $\dot{S}_{gen,T}(t)$ in Eq. (10).

4. The principle of maximum negentropy production (PMNEP)

Statement of PMNEP: '*The isolated system comprising of order co-existing with chaos "will select the path or assemblage of paths out of available paths" that maximizes the negentropy of order at the fastest rate for given constraints*'.

As per LMEP, the isolated system will select the path or assemblage of paths out of available paths that maximizes mean S_{IS} and $\dot{S}_{IS}$ for given constraints (e.g. system inertia). When order exists in chaos, PMNEP states that s_{ni} and $\dot{s}_{ni}$ are maximised, subject to existing constraints that retard their increase. There are the following two possible ways to realise LMEP:

(i) From Eq. (5), because $\dot{S}_{IS}$ is maximised for given constraints (LMEP), $|\dot{s}_{ni}|$ is also maximised for the same constraints when order exists, i.e. LMEP ⇒ PMNEP. This aspect of entropy generation rate was realised much earlier by Carnot for the operation of heat engine, considering work transfer as more orderly than heat transfer. The faster that work is done, more entropy is generated for a given temperature difference between heat source and sink; which in general holds for free energy and its degradation.

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(ii) Alternatively, if $|\dot{s}_{ni}|$ reduces or remains the same, $\dot{s}_{oi}$ increases at a faster rate than $\dot{s}_d$ or is the same as $\dot{s}_d$, respectively; i.e. order *oi* is destroyed into disorder by merging with chaos.

The evolution of dynamic order to superior forms combines the following two important features:

(i) Ability to avoid spontaneously created gradients of field variables across dynamic order $|\mathrm{Grad}(F)|_{oi}$ that exceed the threshold value $[|\mathrm{Grad}(F)|_{oi,\mathrm{thr}}]$. This ability prolongs the existence/survival of dynamic order in chaos, in which, $|\mathrm{Grad}(F)|_{oi}$ is exceeded periodically, randomly, and/or monotonically.
(ii) Ability to generate negentropy at increasing rates by increasing relative ordering, and still surviving by satisfying $|\mathrm{Grad}(F)|_{oi}$ constraint as stated above. Faster rate of negentropy production leads to faster rate of entropy production in chaos, due to existence and evolution of order.

4.1. 'Principle of maximum negentropy production' as the 'law of evolution'

Dynamic order *evolution* as determined by PMNEP encompasses its definitions [7,13], and integrates the notions in the popular evolution postulates [14,15]. The notion of 'mutation' in the evolution postulate refers to *some constraints being revised* in PMNEP. The *selection of path or assemblage of paths out of available paths under given constraints* is the *transformation* of ordered sub-systems sometimes in to superior forms. These constraints are determined jointly by ordered sub-systems and chaos, and are attributed to inertia of order and random contingencies in chaos. Under new set of constraints, ordered sub-systems in conjunction with chaos take noticeably different path/s or their assemblages, for maximising their negentropy. Evolution enables superior order to avoid and/or withstand with increasing probability, the stochastically generated high values of $|\mathrm{Grad}(F)|_{oi}$. This effectively increases $|\mathrm{Grad}(F)|_{oi,\mathrm{thr}}$, thereby improving the ecological niche of the species. The selection of path/s or their assemblages to withstand increasing $|\mathrm{Grad}(F)|_{oi}$ due to s_{ni} increase, enables existence of order. Biological evolutionary features are observed over geological time intervals, as certain constraints are removed and/or minimal thresholds of parameters are reached [35]. A synthesis of review on evolution, and the explanation by PMNEP as the *law of evolution*, is in Table 3. It is seen that the diverse reported interpretations of evolution now fall in proper slot as determined by PMNEP.

4.2. Thermodynamic explanation for co-existence of superior and inferior order

In Eq. (10), the term, $\dot{S}_{\mathrm{gen,T}}(t) = -\sum_{i=1}^{N_e} m_{oi}(t) \cdot \dot{s}_{ni}(t)$, gives entropy generation due to dynamic order existence. The process of evolution leads to dynamic order with higher s_n and $\dot{s}_n$, i.e. superior forms; but all ordered sub-systems are not transformed in to superior forms. This is because superior forms alone do not result in maximum $\dot{S}_{\mathrm{gen,T}}(t)$, and superior forms depend on inferior forms for higher negentropy production. The combination of superior and inferior order (referred as enslaved modes [24]) results in maximum $\dot{S}_{\mathrm{gen,T}}(t)$; hence, they co-exist as per the *slaving principle* (refer Table 2). The slaving principle is now restated as follows: 'Ordered sub-systems that generate higher s_n (and $\dot{s_n}$) tend to enslave those that generate lower s_n (and $\dot{s_n}$).' The universality of this principle is illustrated by the co-existence of biological order as an enslaved mode within cosmological order. The existence of dynamic order and its ability to enslave enables chaos to find its structure that maximises $\dot{S}_{\mathrm{gen,T}}(t)$.

The evolutionary entropy [19] across various existing species is a measure of the variability in the age of reproducing individuals. Evolution increases evolutionary entropy because of the co-existence of superior and inferior species with wide range of life expectancies and bands of reproductive periods.

5. Summary and conclusions

(i) Negentropy is defined as *the specific entropy deficit of ordered sub-system with respect to surrounding chaos*. This definition enables formulation of two exact thermodynamic principles: (*a*) *negentropy principle* (NEP: thermodynamic principle for order existence), and (*b*) *principle of maximum negentropy production* (PMNEP: thermodynamic principle for order evolution).
(ii) The PMNEP encompasses the first principles behind the evolution postulates by *Darwin* and *de Vries*. Perspectives of evolution as reported in literature point to the validity of PMNEP as the *law of evolution* from the physical point of view.
(iii) Evolution of dynamic order in to superior forms is the selection of different path/s or their assemblages, for maximizing the energy flows required to build negentropy.
(iv) When ordered sub-systems exist in chaos, they continue to evolve in to superior forms; i.e. validity of NEP implies the validity of PMNEP, and vice versa also holds. Superior forms have high negentropy, which implies and is implied by high negentropy production rate.
(v) Superior and inferior forms of ordered sub-systems co-exist, because their co-existence generates more entropy than when only superior forms exist. This thermodynamic basis for co-existence explains the increased evolutionary entropy due to evolution.

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Table 3
Review on evolution and role of PMNEP as the universal *Law of evolution*

Author/s	Contribution/s	Interpretation/s based on PMNEP
Spencer [7]	Defined evolution as transformation of less ordered to more ordered states. Biological evolution is part of universal process of evolution	More ordered states generate higher negentropy. Universal evolution is governed by the ubiquitous principle of thermodynamics (PMNEP)
Salthe [13]	Defined evolution as irreversible accumulation of the effects of historical contingency	Historical contingency results in irreversible changes in *paths/their assemblages* in PMNEP
Darwin [14]	Life proceeded to evolve into more complex forms by *natural selection*	*Natural selection* (survival of fittest) is based on PMNEP (survival of species that increase negentropy at fastest rate)
de Vries [15]	Mutations bring about evolution in ordered sub-systems	Sudden changes force order to take noticeably different paths/their assemblages in PMNEP
Odum and Pinkerton [16]	Evolution of ecological systems shows the tendency to maximise rate of energy flows	Maximisation of rate of energy flows leads to maximisation of negentropy increase rate, as stated by PMNEP. This increase in negentropy increases entropy production in chaos in proportion to the amount of gradients of field variables degraded by ecological systems
Runnegar [35]	For evolution, some constraints are removed/minimal thresholds of parameters are reached	Removal of some constraints/attainment of minimal thresholds of parameters enable selection of different paths/their assemblages
Elitzur [17]	Evolution of self-replicating systems is effective in extracting, recording, and processing information about the environment	Extracting, recording, and processing information about the environment by self-replicating systems are highly irreversible processes that increase negentropy
Corbet [20]	Proposed *evolutionary potential* that incorporates entropy effects via EP, which is necessary condition for ordering	Evolutionary potential is the increasing negentropy at fastest possible rate, in PMNEP. Increasing negentropy is a special case of increasing entropy in LMEP and EP
Damiani and Franca [9]	Ordered patterns are fractal-based, which evolve by increasing their complexity during development	These fractal-based patterns are governed by the universal laws of thermodynamics. The increasing complexity [21] is an outcome of path/s taken by ordered patterns in PMNEP
Eigen and Schuster [23]	Proposed hypercycles as a stage of macromolecular evolution, which can follow quasi-species	Hypercycles are ordered cyclic patterns that evolve with the objective of maximising negentropy as per PMNEP
Demetrius [19]	Evolutionary entropy constitutes the operationally valid measure of Darwinian fitness	Evolution increases evolutionary entropy, and superior and inferior order co-exist for maximising entropy production rate in chaos. Variability in the age of reproducing individuals in a population better sustains negentropy production and its maximisation as per PMNEP, which is the physical basis for Darwinian fitness
Ne'eman [21]	Evolution produces ever more ordered matter	More order is produced with the objective of increasing negentropy at the fastest possible rate (as per PMNEP)

(vi) Creation, existence, evolution, and destruction of order, are the paths taken by the local non-equilibrium systems within the isolated system. The objective is maximisation of the global entropy production rate for given constraints, as per LMEP.

(vii) The unification of dynamic order creation, existence, evolution, and destruction, solely by thermodynamic principles, is illustrated by the block diagram in Fig. 3.

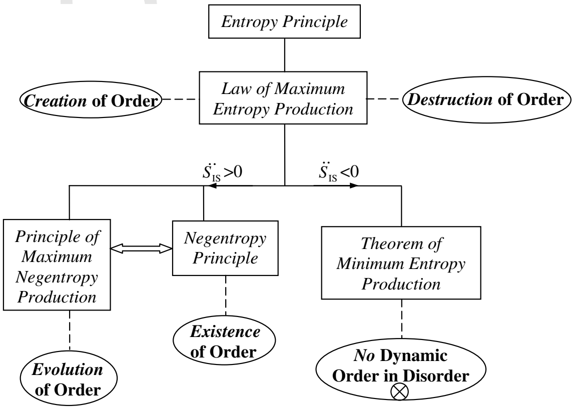

Fig. 3. Principles unifying creation, existence, evolution, and destruction of order.

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